Nutational Stability of an Axisymmetric Body Containing a Rotor

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FOR many satellite and space-probe missions, full three-axis attitude control is not required, and simple (passive) spin stabilization is sufficient to insure satisfactory system performance. In many applications, in fact, the need for stabilization is not dictated by primary mission requirements at all but arises indirectly, perhaps as a means of minimizing or removing thermal and antenna design problems. Spin control is ideal for this type of mission. In other caess, considerable improvement in system performance is afforded, e.g., in a high-altitude communication satellite, increased antenna gain can be obtained using an antenna with a toroidal radiation pattern.

Fully passive techniques for three-axis attitude control have only limited application, and it is generally necessary to employ far more sophisticated, active stabilization methods such as mass expulsion or flywheel control systems. However, there is one active, but fairly simple, three-axis control technique that is a natural extension of spin stabilization. In this technique, control of the third axis (control about the spin axis) is achieved by "despinning" a portion of the spinning spacecraft. That is, instead of the vehicle being a single body, it is constructed of two bodies (I and II in Fig. 1) constrained to rotate about a common axis. Suppose that II is the despun portion. A motor (not shown) is included which can change the relative rate about OZ. If the motor is servo-controlled, say by a sensor mounted on one of the bodies, the rate of body II can be maintained at zero or can be varied to permit a given axis in II, normal to OZ, to track an external reference. Thus, in effect, the despun component is three-axis stabilized by means of a single-axis active controller. Control of spin-axis attitude, if required at all, would normally be by ground command to activate, for example, pulse jets, or, in the case of earth orbiters, a magnetic coil mounted on the spinning component.

A wide range of configurations can be devised which use this basic two-body construction. One application is id the use of a small directional antenna as the despun portion of a spin-stabilized communications satellite. In this application, the spin axis would be maintained normal to the orbit plane, and the antenna controlled to point along local vertical, allowing maximum gain to be obtained.

Another type of single-axis control can be provided if the major part of a spacecraft is despun, and a high-speed internal rotor is employed to provide the gyroscopic stiffness. In fact, this case can be considered a derivative of a three-flywheel system from which two wheels are removed and the third operated about a high bias rate instead of null. Because of the high speed of the internal rotor, spin-axis pointing can again be controlled by ground command.

The two-body configuration does appear to offer definite advantages as a means of achieving simplified three-axis attitude control of spacecraft; for this reason, it has received, and is receiving, serious attention in the industry. Indeed, one spacecraft of this construction, OSO I (built by Ball Brothers for NASA), was launched over two years ago and has operated successfully in orbit. In this paper the nutational stability of the two-body system is considered, and the constraints that the requirement for stability places on the moments of inertia of the configuration are established.

Received June 17, 1964.

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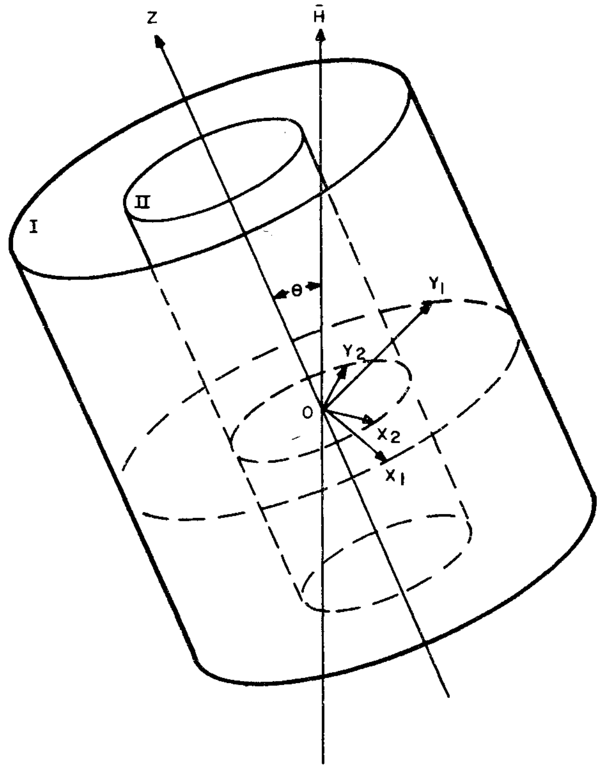

SYMBOLS:
OX_1, OY_1, OZ ARE AXES FIXED IN BODY I
OX_2, OY_2, OZ ARE AXES FIXED IN BODY II
$\bar{H}$ IS THE ANGULAR MOMENTUM VECTOR
θ IS THE NUTATION HALF ANGLE

NOTES:
1. OX_1, IS NORMAL TO OY_1, AND OX_2 IS NORMAL TO OY_2
2. OX_1, OY_1, OX_2, AND OY_2 ARE NORMAL TO OZ

Fig. 1 Two-body configuration.

Stability Control

We first recall a standard result of classical mechanics: steady, stable rotational motion of a rigid body under no external forces is possible only if the axis of rotation is a principal axis of least or greatest inertia. If the motion is slightly disturbed from an initial state of steady rotation about either one of these axes, the resulting motion of the principal axis is bounded with the angle between the axis and the angular momentum vector never exceeding a certain value determined by the magnitude and direction of the disturbance. Such motion is called nutation or free precession. In the case of an axisymmetric body spinning about the axis of symmetry, a disturbance causes this axis to describe a right circular cone about the angular momentum vector. On the other hand, if a body is not rigid and energy is dissipated by the cyclic forces acting on it while under nutation, then only motion about the axis of maximum inertia is stable, for, in this state, the kinetic energy is a minimum. In this case, energy dissipation causes the axis of maximum inertia to move into coincidence with the angular momentum vector, thus removing any nutation initially present. Thomson and Reiter[1] established this condition for stability for the general case of an unspecified damper, although most standard dynamics texts derive the result for particular cases.

The following analysis is restricted to the case of two coaxial, axisymmetric bodies, at least one of which is spinning and where only one of the bodies is taken to be energy dissipative, that is, to contain a damper. The majority of satellite applications would fall into this class, except perhaps for the last assumption. The determination of stability conditions for the case of distributed damping requires a knowledge of the rates of energy dissipation for each body. Since, in this situation, a complete analysis of the dynamics of the system would probably be necessary, there would appear to be little value in obtaining generalized conditions. However, in most satellites, dissipation is obtained solely from a damping device designed specifically for the removal of nutation. The model discussed here is appropriate to this case.

The two-body system is said to be stable if the common axis of the two bodies tends to move into coincidence with the invariant angular momentum vector **H** ($\dot{\theta}$ negative) under the action of the damping forces existing during nutation. Motion is possible for which the nutation half-angle θ is greater than $\pi/2$, and so, to avoid ambiguity, the system is described as stable only if the angular momentum and spin axes are in the same sense (that is, if $\theta = 0$) at equilibrium. It follows that, for stability, the energy must be a monotonic increasing function of the nutation angle θ.

The conditions for stability are perhaps surprising in that they depend on the location of the damper. In most satellites, one body would be stationary or, more probably, rotating slowly to track the local vertical or the sun. In such cases, the conditions for stability are very nearly as follows: 1) if the damper is located on the spinning component, the polar moment of inertia of this component alone must be greater than the total transverse moment of inertia, or 2) if the damper is located on the stationary component, there is no constraint, the system being stable for all ratios of the inertias. The following analysis does assume the centers of gravity coincident. However, the conditions hold even if the centers of gravity are not coincident, since it is understood that, in this case, the total transverse inertia is not equal to the sum of the transverse inertias of the individual components.

Condition 1 is hard to satisfy in actual design without introducing extendable arms on the spinning body. It will be apparent to the reader, therefore, that as a general rule a stator-mounted damper should be employed on a two-body satellite wherever possible, since, in this case, there is virtually no restriction on the inertia ratios. Thus, for systems employing the two-body configuration as a means of simplified attitude control, there is the additional advantage in that sometimes serious vehicle inertia constraints can be removed by suitable choice of damper location.

Analysis

The kinetic energy (T_k) of the two-body system (Fig. 1) is given by

$$2T_k = A_1\omega_{1X}^2 + A_1\omega_{1Y}^2 + C_1\omega_{1Z}^2 + A_2\omega_{2X}^2 + A_2\omega_{2Y}^2 + C_2\omega_{2Z}^2$$

where

$\omega_{1X}, \omega_{1Y}, \omega_{1Z}$ = angular velocities of body I about OX_1, OY_1, OZ

$\omega_{2X}, \omega_{2Y}, \omega_{2Z}$ = angular velocities of body II about OX_2, OY_2, OZ

A_1, A_1, C_1 = moments of inertia of body I about OX_1, OY_1, OZ

A_2, A_2, C_2 = moments of inertia of body II about OX_2, OY_2, OZ

However, the components of transverse angular velocity are constrained to be equal, that is,

$$\omega_{1X}^2 + \omega_{1Y}^2 = \omega_{2X}^2 + \omega_{2Y}^2 \equiv \omega_{XY}^2$$

then

$$2T_k = A\omega_{XY}^2 + C_1\omega_{1Z}^2 + C_2\omega_{2Z}^2 \tag{1}$$

where $A = A_1 + A_2$. Also, from Fig. 1,

$$H\cos\theta = C_1\omega_{1Z} + C_2\omega_{2Z} \text{ and } H\sin\theta = A\omega_{XY} \tag{2}$$

where H is the magnitude of the angular momentum vector **H**, and θ is the nutation half amplitude.

From Eqs. (1) and (2) and their derivatives, two alternative expressions for the time rate of change of kinetic energy can be found:

$$\dot{T}_k = \frac{H\sin\theta}{A}\dot{\theta}[(C_2 - A)\omega_{2Z} + C_1\omega_{1Z}] - C_1\dot{\omega}_{1Z}(\omega_{2Z} - \omega_{1Z}) \tag{3}$$

and

$$\dot{T}_k = \frac{H\sin\theta}{A}\dot{\theta}[(C_1 - A)\omega_{1Z} + C_2\omega_{2Z}] + C_2\dot{\omega}_{2Z}(\omega_{2Z} - \omega_{1Z}) \tag{4}$$

Motor Absent

To simplify the development, consider first an idealized vehicle with frictionless bearings between the bodies and where no motor is included to maintain relative motion. Equations (2) show that, as the nutation angle varies, the angular momentum component about the spin axis increases (or decreases) at the expense of the transverse component. However, the body which does not contain a damper must obey Euler's equations for rigid body motion. Suppose this to be body I. Then Euler's equations indicate that, however θ and the spin rate of body II vary under the action of the damper mounted on II, the spin rate (ω_{1Z}) of body I, which is rigid, must remain constant since there is no applied torque about the spin axis. Hence, the last term in Eq. (3) vanishes, and, from the same equation, it is seen that for $\dot{T}_k$ and $\dot{\theta}$ to be of the same sign (the condition for stability) the expression in brackets must be positive for $0 < \theta < \pi$.

Therefore, for the case of a damper mounted on body II, the required condition is

$$(C_2 - A)\omega_{2Z} + C_1\omega_{1Z} > 0 \tag{5}$$

Similarly, from (4) the stability condition for a damper mounted on body I is

$$(C_1 - A)\omega_{1Z} + C_2\omega_{2Z} > 0 \tag{6}$$

Equations (5) and (6) are the exact relations from which approximate conditions 1 and 2 given earlier are derived. The conditions themselves depend on the nutation amplitude, e.g., Eq. (5) can be rewritten

$$H\cos\theta\left(\frac{1}{A} - \frac{1}{C_2}\right) + \frac{C_1\omega_{1Z}}{C_2} > 0$$

For certain values of the constants, the left-hand side of this inequality can change sign in the interval $0 < \theta < \pi$. In other words, the region of stability can be limited. It is important in practice, therefore, to insure, by a suitable choice of satellite parameters, that this region includes the maximum anticipated initial nutation amplitude.

Motor Included

With a motor included, both ω_{1Z} and ω_{2Z} can be varied by applying a torque through the motor. In particular, it is possible to maintain a constant spin rate for the body containing the damper. It would appear, therefore, from Eqs. (3) and (4) that the conditions for stability in this case depend on which body is maintained at constant rate and not on the location of the damper. The fallacy of this deduction lies in the fact that, with the motor absent, the kinetic energy is the total energy of the system, whereas here the motor is supplying (or removing) energy by varying the relative spin rate of the bodies. In other words, the motor is equivalent to an energy source or a sink which must be taken into account in the analysis. Of course, energy is also supplied by the motor to overcome bearing friction, but as this fraction of the energy is dissipated directly as heat, it need not be considered.

With a motor included, we have a modified condition for stability: the rate of change of total system energy must be of the same sign as $\dot{\theta}$. This rate is the sum of the rate of change of kinetic energy and the rate of motor energy "transfer," the latter being positive if energy is removed by the motor. Neglecting friction, the spin rate of the body not containing the damper can be changed only by the application of motor torque. The rate at which energy is transferred is the product of this torque and the relative spin rate. Again, suppose that body I is rigid. Then the motor torque is equal to $C_1\dot{\omega}_{1Z}$, and the rate at which energy is supplied or removed by the motor ($\dot{T}_M$) is given by

$$\dot{T}_M = C_1\dot{\omega}_{1Z}(\omega_{2Z} - \omega_{1Z}) \tag{7}$$

Note that $\dot{T}_M \neq C_2\dot{\omega}_{2Z}(\omega_{2Z} - \omega_{1Z})$ in magnitude, as body II is also losing (or gaining) spin momentum because of the presence of the damper.

Assume that $\omega_{2Z} > \omega_{1Z}$. Then, if $\dot{\omega}_{1Z}$ is positive, there is a net reduction in kinetic energy due to the action of the motor. Hence, the total rate of change of system energy is the sum of the rates given by Eqs. (3) and (7) and the condition for stability again reduces to that given by Eq. (5) for the no-motor case. If body II is rigid,

$$\dot{T}_M = C_2\dot{\omega}_{2Z}(\omega_{2Z} - \omega_{1Z}) \tag{8}$$

and, for $\omega_{2Z} > \omega_{1Z}$, kinetic energy is increased for positive $\dot{\omega}_{2Z}$. The rate of change of total system energy can be obtained in this case by taking the difference of Eqs. (4) and (8), which leads to a condition for stability given by Eq. (6). We conclude, therefore, that the presence of the motor does not in any way affect the result and that the conditions for stability in Eqs. (5) and (6) depend only on the location of the damper. If one body is stationary or rotating very slowly compared with the other, the conditions reduce to those given earlier in the paper.

Reference

[1] Thomson, W. T. and Reiter, G. S., "Attitude drift of space vehicles," J. Astronaut. Sci. 7, 29–34 (1960).

Alouette Topside Sounder Satellite: Experiments, Data, and Results

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Introduction

THE Alouette satellite (1962 Beta Alpha One) was launched 0605 GMT September 29, 1962, from Vandenburg Air Force Base, Calif. into a nearly circular orbit with an inclination of 80.5° and 1000 km above the earth. The satellite was designed and constructed in Canada at the Defence Research Telecommunications Establishment and launched into orbit on a Thor-Agena B.

In the first ten months of its operation, the satellite received and executed over 10,000 commands in 2000 hr of operation of the four experiments aboard. In this time, there were no equipment failures. The sole observable degradation was in reduced power output (to 63% of the output at launch) of the solar cells due largely to energetic particle damage from the artificial radiation belt produced by the Starfish test of July 9, 1962.

Figure 1 shows the Alouette spacecraft ready for launch. It is a flattened sphere in shape, covered with about 6500 solar cells, and weighs 319 lb. Its most distinguishing feature is the pair of crossed antennas, one 150 ft and the other 75 ft from tip to tip. Designed with reliability as a prime consideration, the spacecraft has been described in detail elsewhere.[2,12]

Alouette carries three novel experiments, a sweep-frequency (0.5 to 11.5 Mc/sec) topside sounder, a receiver (part of the sounder) capable of monitoring cosmic radio noise, and a very-low-frequency (VLF) receiver. A group of six counters for energetic particles makes the fourth experiment. This paper will discuss these four experiments and summarize briefly some results obtained.

Topside Sounder

The term "topside sounder" has been coined to describe an experiment in which an ionosonde is placed above the *F*-layer peak of the ionosphere. Figure 2 shows diagramatically the electron number density distribution below 1000 km and the complementary fashion in which a topside and a bottomside ionosonde can be used to determine this distribution. (An ionosonde measures echo range of a radio pulse of appropriate frequency.)

The Alouette topside sounder enables the topside distribution to be measured, under favourable conditions, approximately every 100 km when the satellite is operating. The NASA Minitrack chain of stations in North and South America, along with telemetry stations in Canada, and the South Atlantic station operated by Great Britain, permit measurements to be made near the 75°W meridian nearly continuously from 80°N to 80°S latitude. After a somewhat lengthy analysis procedure, a cross section of the ionosphere, as shown in Fig. 3, can be produced.

Figure 3 shows contours of constant electron density, in megacycles, as a function of height and latitude. It also demonstrates the very comprehensive measurement of the ionosphere in depth which is possible from a topside sounder satellite.

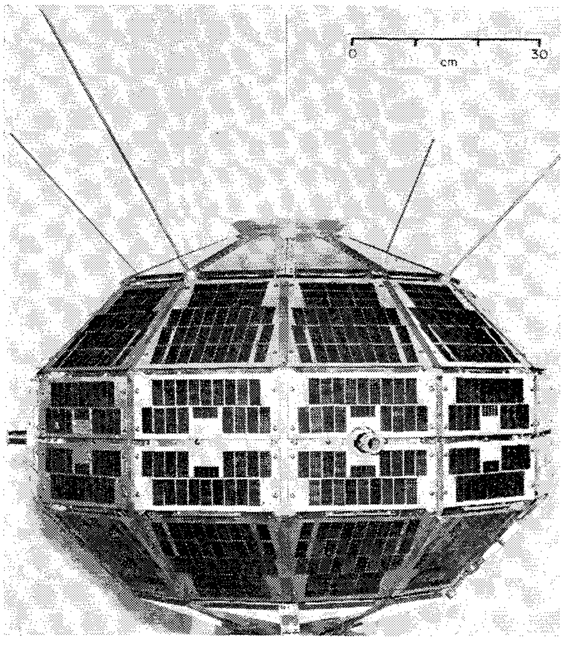

Fig. 1 Alouette spacecraft with sounding antennas retracted in launch position (after Molozzi).

Presented as Preprint 63-469 at the AIAA/CASI/RAeS 9th Anglo-American Conference, Cambridge, Mass., October 16–18, 1963, and Montreal, Canada, October 21–24, 1963; revision received September 3, 1964. Published with the permission of The Royal Aeronautical Society and the Canadian Aeronautics and Space Institute.

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